

Gap equation in scalar field theory at finite temperature

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Abstract

We investigate the two-loop gap equation for the thermal mass of hot massless $g^2\phi^4$ theory and find that the gap equation itself has a non-zero finite imaginary part. This indicates that it is not possible to find the real thermal mass as a solution of the gap equation beyond g^2 order in perturbation theory. We have solved the gap equation and obtain the real and the imaginary part of the thermal mass which are correct up to g^4 order in perturbation theory.

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It is well-known [1] that if a theory contains massless bosonic field such as, QCD or massless scalar theory with $g^2\phi^4$ interaction, then at very high temperature (T) the perturbative computations beyond certain order of coupling constant are afflicted with infrared (IR) singularities. In the case of massless $g^2\phi^4$ theory, the one loop contribution to two point function shows that the fields are screened and the screening mass (Debye mass) is found to be of order gT [2, 3]. However, the result of two loop corrections to it is found to be IR divergent. A natural way of avoiding this IR divergences in the two loop computation is to use the dynamically generated one loop thermal mass gT as the lower cut-off of the momentum integration. As a result one finds the appearance of a new g^3 order correction to two point function which, although consistent with the spirit of perturbation theory, has not been predicted from the usual perturbative expansion in powers of coupling constant (g) at zero temperature. In addition to that there are infinite number of higher order diagrams that contribute to this particular g^3 order which in turn is the signature of the break-down of usual perturbation theory at very high temperature. Moreover, it also suggests that one has to resum this infinite number of diagrams to correctly calculate this g^3 order contribution [2, 4, 5] to two-point function.

However remaining within the framework of perturbation theory one can in principle be able to calculate the thermal mass to any order of coupling constant by solving the gap equation [6, 7]. In order to obtain the gap equation, the functional integral formulation may be used, and we shall briefly discuss this method following Jackiw *et. al.* [7]. Consider a massless $g^2\phi^4$ theory in d dimension ($d = 4 - 2\epsilon$) described by the lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mu^{2\epsilon}\frac{g^2}{4!}\phi^4, \quad (1)$$

and the partition function is given by

$$Z = \int \mathcal{D}\phi e^{iS(\phi)}, \quad (2)$$

where $S(\phi) = \int d^d x \mathcal{L}(\phi(x))$. We introduce a loop counting parameter l and write down Z as

$$Z = \int \mathcal{D}\phi e^{\frac{i}{l}S(\sqrt{l}\phi)}. \quad (3)$$

In the usual perturbation theory we separate the quadratic part of $S(\phi)$ and expand the exponential of the remainder in powers of l . To obtain a gap equation for a possible mass m , we add and subtract $S_m = -\frac{m^2}{2} \int d^d x \phi^2(x)$, which of course changes nothing (at least in the classical level, the equation of motion remains the same).

$$S = S + S_m - S_m \quad (4)$$

We recognize the loop expansion by expanding $S + S_m$ in the usual way, but taking $-S_m$ as contributing at one loop higher. These can be accounted systematically by replacing Eq. (4) with an effective action S_l .

$$S_l = \frac{1}{l} [S(\sqrt{l}\phi) + S_m(\sqrt{l}\phi) - S_m(\sqrt{l}\phi)] . \quad (5)$$

Starting from this effective action the self energy Σ of the complete propagator can be calculated to any order in l and set $l = 1$ at the end of the calculation. The gap equation is obtained by demanding that Σ does not shift the mass m , i.e.,

$$\Sigma(p) |_{p^2=m^2} = 0 \quad (6)$$

Furthermore, in order to get a real solution of m from Eq. (6) one has to ensure that the imaginary part of $\Sigma(p)$ at $p^2 = m^2$ is zero.

We apply the above mentioned ideas to $g^2\phi^4$ theory (described by Eq. (1)) at finite temperature using real time formalism. Let us recall that in the real time formalism [8, 9] the thermal propagator has a 2×2 matrix structure, the $1 - 1$ component of which refers to the physical field, the $2 - 2$ component to the corresponding ghost field, with the off-diagonal $1 - 2$ and $2 - 1$ components mixing them. The propagator used here is given by

$$\begin{pmatrix} D_{11}(K) & D_{12}(K) \\ D_{21}(K) & D_{22}(K) \end{pmatrix} = \begin{pmatrix} \Delta_0(K) + \Delta_\beta(K) & \bar{\Delta}_\beta(K) \\ \bar{\Delta}_\beta(K) & \Delta_0^*(K) + \Delta_\beta(K) \end{pmatrix} , \quad (7)$$

where $\Delta_0(K)$ is the usual Feynman propagator at zero temperature

$$\Delta_0(K) = \frac{i}{K^2 - m^2 + i\epsilon} . \quad (8)$$

Here Δ_β and $\bar{\Delta}_\beta$ are finite temperature corrections to the zero temperature propagator where

$$\Delta_\beta(K) = 2\pi\delta(K^2 - m^2)n_B(|K_0|), \quad \bar{\Delta}_\beta(K) = 2\pi\delta(K^2 - m^2)\exp\left(\frac{\beta|K_0|}{2}\right)n_B(|K_0|), \quad (9)$$

with the Bose-Einstein factor $n_B(K_0) = 1/(e^{\beta|K_0|} - 1)$ ($\beta = 1/T$). The complete self energy is given by the expression [9, 10]:

$$\text{Re}\Sigma(p) = \text{Re}\Sigma_{11}(p), \quad \text{Im}\Sigma(p) = \frac{i}{2} \frac{e^{\beta p_0} - 1}{e^{\beta p_0/2}} \Sigma_{12}(p), \quad (10)$$

where Σ_{11} and Σ_{12} are the self energy of the $1 - 1$ component and $1 - 2$ component, respectively.

The graphs that are contributing to $\Sigma(p)$ up to two-loop order are depicted in Fig. 1. The contributions to $\text{Re}\Sigma(p)$ coming from these graphs, although in a different context, have already been computed in Refs. [2, 3, 6, 11]. We find that the real part of self energy in the limit $p_0 = m$ and $\mathbf{p} \rightarrow 0$ is given by

$$\begin{aligned} \text{Re}\Sigma(p_0 = m, \mathbf{p} \rightarrow 0) = & -y^2 T^2 + \hat{g}^2 T^2 \left\{ -\pi y + y^2 \sum_{n=1}^{\infty} B_n(y) - \sum_{n=1}^{\infty} A_n(y) \right\} \\ & + \hat{g}^4 T^2 \left\{ 2\pi^2 + a_1 y - 2\pi y \ln y^2 - a_2 y^2 + a_3 y^2 \ln y^2 \right. \\ & - \frac{y^2}{4} (\ln y^2)^2 - (a_4 + \ln y^2 - \frac{\pi}{y}) \sum_{n=1}^{\infty} A_n(y) \\ & - (a_5 y^2 - 2\pi y) \sum_{n=1}^{\infty} B_n(y) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_m(y) B_n(y) \\ & \left. + 4[X(y) + I(y)] \right\} + 0(\hat{g}^5), \end{aligned} \quad (11)$$

where $y = \frac{m}{T}$, $\hat{g} = \frac{g}{4\pi}$, $a_1 = 15.64 - 21.99 \ln(\frac{T}{\mu})$, $a_2 = 26.07 - 3.46 \ln(\frac{T}{\mu}) - 2(\ln(\frac{T}{\mu}))^2 - \frac{1}{6}\delta_1 + \frac{1}{2}\delta_2$, $a_3 = 1.62 + \frac{1}{3} \ln(\frac{T}{\mu})$, $a_4 = 2.54$, $a_5 = 0.98 + \ln(\frac{T}{\mu})$, $A_n(y) = \frac{1}{n}[8\pi^2 n^2(1 + \frac{y^2}{4\pi^2 n^2})^{\frac{1}{2}} - y^2]$, $B_n(y) = \frac{1}{n}[(1 + \frac{y^2}{4\pi^2 n^2})^{-\frac{1}{2}} - 1]$,

$$\delta_1 = \int_0^1 dx_1 \int_0^1 dx_2 (1 - x_2) \ln [(1 - x_2 + \frac{x_2}{(1-x_1)x_1}) - x_2(1 - x_2)],$$

$$\delta_2 = \int_0^1 dx_1 \int_0^1 dx_2 (\frac{1}{x_2} - 1) \ln [(1 - x_2 + \frac{x_2}{(1-x_1)x_1}) - x_2(1 - x_2)],$$

$$X(y) = y^2 \int_1^\infty \frac{dt}{(e^{ty} - 1)} t \ln(t + \sqrt{t^2 - 1}),$$

$$I(y) = y^2 \int_1^\infty \frac{dt_2}{(e^{yt_2} - 1)} \text{PP} \int_1^\infty \frac{dt_1}{(e^{yt_1} - 1)} \ln \left[\frac{\sqrt{t_2^2 - 1} - \sqrt{t_1^2 - 1}}{\sqrt{t_2^2 - 1} + \sqrt{t_1^2 - 1}} \right].$$

The only graph that will contribute to the $\text{Im}\Sigma(p)$ is Fig. 1(h) and its contribution to $\Sigma_{12}(p)$ is given as

$$\begin{aligned} -i\Sigma_{12}(p) = & \frac{g^4}{6} \int \frac{d^4 K_1}{(2\pi)^4} \frac{d^4 K_2}{(2\pi)^4} [\bar{\Delta}_\beta(K_1) \bar{\Delta}_\beta(K_2) \bar{\Delta}_\beta(p - K_1 - K_2)] \\ = & \frac{g^4}{6(2\pi)^5} \int d^3 k_1 d^3 k_2 \frac{1}{4E_{k_1} E_{k_2}} n_B(E_{k_1}) n_B(E_{k_2}) \\ & n_B(E_{p-k_1-k_2}) e^{\beta E_{k_1}/2} e^{\beta E_{k_2}/2} e^{\beta E_{p-k_1-k_2}/2} \\ \times & \left\{ \delta \left[(p_0 - E_{k_1} - E_{k_2})^2 - E_{p-k_1-k_2}^2 \right] \right. \\ & + \delta \left[(p_0 - E_{k_1} + E_{k_2})^2 - E_{p-k_1-k_2}^2 \right] \\ & + \delta \left[(p_0 + E_{k_1} - E_{k_2})^2 - E_{p-k_1-k_2}^2 \right] \\ & \left. + \delta \left[(p_0 + E_{k_1} + E_{k_2})^2 - E_{p-k_1-k_2}^2 \right] \right\}, \end{aligned} \quad (12)$$

where $E_{k_1} = \sqrt{\mathbf{k}_1^2 + m^2}$, $E_{k_2} = \sqrt{\mathbf{k}_2^2 + m^2}$, and $E_{p-k_1-k_2} = \sqrt{(\mathbf{p} - \mathbf{k}_1 - \mathbf{k}_2)^2 + m^2}$. In the $p_0 = m$ and $\mathbf{p} \rightarrow 0$ limit, the expression for $-i\Sigma_{12}(p)$, becomes,

$$\begin{aligned}
-i\Sigma_{12}(p_0 = m, \mathbf{p}) &= \frac{g^4}{6(2\pi)^5} \int d^3k_1 d^3k_2 \frac{1}{E_{k_1} E_{k_2}} n_B(E_{k_1}) n_B(E_{k_2}) \\
&\quad n_B(E_{k_1+k_2}) e^{\beta E_{k_1}/2} e^{\beta E_{k_2}/2} e^{\beta E_{k_1+k_2}/2} \\
&\quad \times \left\{ \delta[(m - E_{k_1} - E_{k_2})^2 - E_{k_1+k_2}^2] \right. \\
&\quad \quad + \delta[(m - E_{k_1} + E_{k_2})^2 - E_{k_1+k_2}^2] \\
&\quad \quad + \delta[(m + E_{k_1} - E_{k_2})^2 - E_{k_1+k_2}^2] \\
&\quad \quad \left. + \delta[(m + E_{k_1} + E_{k_2})^2 - E_{k_1+k_2}^2] \right\} \\
&\tag{13}
\end{aligned}$$

If \mathbf{k}_1 makes an angle θ with the k_{2z} direction then, the above expression becomes

$$\begin{aligned}
-i\Sigma_{12}(p_0 = m, \mathbf{p} \rightarrow 0) &= \frac{g^4}{48(2\pi)^4} \int \frac{d^3k_1}{k_1 E_{k_1}} \int_0^\infty \frac{k_2 dk_2}{E_{k_2}} \int_{-1}^{+1} dx n_B(E_{k_1}) n_B(E_{k_2}) \\
&\quad n_B(E_{k_1+k_2}) e^{\beta E_{k_1}/2} e^{\beta E_{k_2}/2} \\
&\quad \times \left\{ \delta(x - x_1) \theta(1 - |x_1|) + \delta(x - x_2) \theta(1 - |x_2|) \right. \\
&\quad \quad \left. + \delta(x - x_3) \theta(1 - |x_3|) + \delta(x - x_4) \theta(1 - |x_4|) \right\} \\
&\tag{14}
\end{aligned}$$

where

$$\begin{aligned}
x &= \cos\theta \\
x_1 &= \frac{(m - E_{k_1} - E_{k_2})^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2 - m^2}{2k_1 k_2}; x_2 = \frac{(m - E_{k_1} + E_{k_2})^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2 - m^2}{2k_1 k_2} \\
x_3 &= \frac{(m + E_{k_1} - E_{k_2})^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2 - m^2}{2k_1 k_2}; x_4 = \frac{(m + E_{k_1} + E_{k_2})^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2 - m^2}{2k_1 k_2}
\end{aligned}
\tag{15}$$

Now we have to evaluate the range of integrations over \mathbf{k}_1 and \mathbf{k}_2 using this theta functions.

1) $\theta(1 - |x_1|) = \theta(1 - x_1)\theta(1 + x_1)$. Therefore $x_1 \leq 1$ implies

$$\sqrt{E_{k_1} - m} \sqrt{E_{k_2} - m} \left(\sqrt{(E_{k_1} + m)(E_{k_2} + m)} - \sqrt{(E_{k_1} - m)(E_{k_2} - m)} \right) \geq 0,$$

and it is trivially satisfied for positive values of k_1 and k_2 . Similarly $x_1 \geq -1$ implies

$$(E_{k_1} - m)(E_{k_2} - m) + k_1 k_2 \geq 0,$$

and is also satisfied by both of k_1 and k_2 . Therefore, this theta function does not put any extra constraint on the integration range of k_1 and k_2 .

2) $\theta(1 - |x_2|) = \theta(1 - x_2)\theta(1 + x_2)$. $x_2 \leq 1$ implies

$$(E_{k_1} - m)(E_{k_2} - m) + k_1 k_2 \geq 0,$$

and it is trivially satisfied for positive values of both of k_1 and k_2 . Next $x_2 \geq -1$ implies

$$k_2(k_2 - k_1) \geq 0.$$

Therefore it gives $\theta(1 - |x_2|) = \theta(k_2 - k_1)$.

3) $\theta(1 - |x_3|) = \theta(1 - x_3)\theta(1 + x_3)$. Proceeding in the similar fashion one gets $\theta(1 - |x_3|) = \theta(k_1 - k_2)$.

4) $\theta(1 - |x_4|) = \theta(1 - x_4)\theta(1 + x_4)$. $x_4 \leq 1$ implies

$$k_2(k_2 + k_1) \leq 0,$$

and this relation is not been satisfied for positive values of k_1 and k_2 . Therefore this theta function will not give any contribution to $-i\Sigma_{12}(p_0 = m, \mathbf{p} \rightarrow 0)$.

Using this theta function constraints we ultimately get,

$$\begin{aligned} -i\Sigma_{12}(p_0 = m, \mathbf{p} \rightarrow 0) &= \frac{g^4}{24(2\pi)^3} \int_0^\infty \frac{dk_1 k_2}{E_{k_1}} \int_0^\infty \frac{dk_2 k_2}{E_{k_2}} \\ &\quad n_B(E_{k_1}) n_B(E_{k_2}) e^{\beta E_{k_1}/2} e^{\beta E_{k_2}/2} \\ &\times \left\{ n_B(E_{k_1} + E_{k_2} - m) e^{\beta(E_{k_1} + E_{k_2} - m)/2} \right. \\ &\quad + \theta(k_2 - k_1) n_B(E_{k_2} - E_{k_1} + m) e^{\beta(E_{k_2} - E_{k_1} + m)/2} \\ &\quad \left. + \theta(k_1 - k_2) n_B(E_{k_1} - E_{k_2} + m) e^{\beta(E_{k_1} - E_{k_2} + m)/2} \right\} \quad (16) \end{aligned}$$

Now if we interchange k_1 and k_2 in the last term of the above expression it becomes identical to the second term, and we get,

$$\begin{aligned} -i\Sigma_{12}(p_0 = m, \mathbf{p} \rightarrow 0) &= \frac{g^4}{24(2\pi)^3} \left\{ \int_0^\infty \frac{dk_1 k_1}{E_{k_1}} \int_0^\infty \frac{dk_2 k_2}{E_{k_2}} e^{\beta(E_{k_1} + E_{k_2})} e^{\beta m/2} n_B(E_{k_1} + E_{k_2} - m) \right. \\ &\quad + \int_0^\infty \frac{dk_2 k_2}{E_{k_2}} \int_0^{k_2} \frac{dk_1 k_1}{E_{k_1}} e^{\beta E_{k_2}} e^{\beta m/2} n_B(E_{k_2} - E_{k_1} + m) \left. \right\} \\ &\times n_B(E_{k_1}) n_B(E_{k_2}) \quad (17) \end{aligned}$$

Finally the $\text{Im}\Sigma(p_0 = m, \mathbf{p} \rightarrow 0)$ takes the following form:

$$\begin{aligned}\text{Im}\Sigma(p_0 = m, \mathbf{p} \rightarrow 0) &= \frac{i}{2} \frac{e^{\beta m} - 1}{e^{\beta m/2}} \Sigma_{12}(p_0 = m, \mathbf{p} \rightarrow 0) \\ &= -\frac{2\pi}{3} \hat{g}^4 T^2 J(y)\end{aligned}\quad (18)$$

where $\bar{n}_B(x) = 1/(e^x - 1)$ and

$$J(y) = 2 \left(\ln(1 - e^{-y}) \right)^2 - 3 \int_y^\infty dx \ln(1 - e^{-x}) - 2 \int_y^\infty dx (x - y) \bar{n}_B(x + y) \quad (19)$$

Therefore it is evident from Eqs. (11) and (18) that $\Sigma(p) |_{(p_0 = m, \mathbf{p} \rightarrow 0)}$ is complex. Consequently, the thermal mass(m) that we may obtain by solving the gap equation $\Sigma(p) |_{(p_0 = m, \mathbf{p} \rightarrow 0)} = 0$ has an imaginary part. However, it is clear from Eq. (10) that due to a multiplicative factor $(e^{\beta p_0} - 1)$, $\text{Im}\Sigma(p)$ is zero at $p_o = |\mathbf{p}|$ and $|\mathbf{p}| \rightarrow 0$ limit and hence $\Sigma(p)$ is real in this limit.

At very high temperature $y \ll 1$ and the real and imaginary part of the self energy in this high temperature limit takes the following form:

$$\begin{aligned}\text{Re}\Sigma(y) &= -T^2 y^2 + \hat{g}^2 T^2 \left\{ \frac{2\pi^2}{3} - \pi y \right\} + \hat{g}^4 T^2 \left\{ 3.7\pi^2 - \frac{2\pi^3}{3} \frac{1}{y} - b_1 y - b_2 y^2 \right. \\ &\quad \left. + \left(\frac{4\pi^2}{3} + 2\pi \right) \ln y^2 - b_3 y^2 \ln y^2 \right\} + 0(\hat{g}^5)\end{aligned}\quad (20)$$

and

$$\text{Im}\Sigma(y) = \hat{g}^4 T^2 \left\{ \frac{2\pi^3}{9} + 7.9y - \frac{\pi}{2} y^2 + \frac{\pi}{3} y \ln y^2 - \frac{\pi}{3} (\ln y^2)^2 \right\} + 0(\hat{g}^5) \quad (21)$$

where, $b_1 = 37.12 - a_1$, $b_2 = a_2 - 7.36$ and $b_3 = 2.7 - a_3$. Therefore the gap equation in this limit is

$$\text{Re}\Sigma(y) + i\text{Im}\Sigma(y) = 0. \quad (22)$$

Since y is complex, we set $y^2 = y_R^2 + iy_I^2$ and substitute it in the high temperature approximated gap equation(22) to obtain the following two equations:

$$y_R^2 = f_R(y_R, y_I) \quad (23)$$

$$y_I^2 = f_I(y_R, y_I) \quad (24)$$

where y_R and y_I are the real and the imaginary part of the thermal mass respectively in the high temperature limit and the explicit form of f_R and f_I are given in the appendix. The eqn.(23) and eqn.(24) are transcendental equations in two variables y_R and y_I and we can solve this equation by the method of iteration[12] assuming that our solution is correct up to order \hat{g}^4 . We have started first with an approximate values of a pair of roots.

$$y_R^{(0)2} = \frac{2\pi^2}{3}\hat{g}^2 + 3.7\pi^2\hat{g}^4, \quad y_I^{(0)2} = \frac{2\pi^3}{9}\hat{g}^4$$

After third iteration we find that the improved pair of roots are equal to the pair of roots obtained after second iteration. Therefore at very high temperature the real and the imaginary part of the thermal mass up to \hat{g}^4 order is

$$\begin{aligned} m_R^2 &= \frac{2\pi^2}{3}T^2\hat{g}^2 - \sqrt{\frac{8}{3}}\pi^2T^2\hat{g}^3 + \left[3.7\pi^2 + \left(\frac{4\pi^2}{3} + 2\pi\right)\ln\left(\frac{2\pi^2}{3}\right)\right]T^2\hat{g}^4 \\ &\quad - \left(\frac{8\pi^2}{3} + 4\pi\right)T^2\hat{g}^4\ln\left(\frac{1}{\hat{g}}\right) + 0(\hat{g}^5) \end{aligned} \quad (25)$$

and

$$m_I^2 = \left[\frac{2\pi^3}{3} - \frac{\pi}{3}\left(\ln\left(\frac{2\pi^2}{3}\right)\right)^2\right]T^2\hat{g}^4 + \frac{4\pi}{3}T^2\ln\left(\frac{1}{\hat{g}}\right) - \frac{4\pi}{3}T^2\left(\ln\left(\frac{1}{\hat{g}}\right)\right)^2 + 0(\hat{g}^5) \quad (26)$$

We see that up to g^4 order both m_R^2 and m_I^2 are independent of the ultraviolet scale μ used in the theory. However from the structure of f_R and f_I , it is evident that beyond g^4 order they ought to be μ -dependent. The result of m_R^2 up to order g^3 matches with that obtained by Parwani[13], however his g^4 order term is μ dependent. As a result the two loop real thermal mass obtained in [13] becomes unstable below some characteristics scale of the order of $g^{4/3}T$.

In this letter we have studied the finite temperature gap equation in massless $g^2\phi^4$ theory and its nature of solutions. We find that the thermal mass up to two-loop order which one may obtain self-consistently by solving this equation is complex. In the massless $g^2\phi^4$ theory the typical one-loop thermal mass is of order gT and the expected contribution to the mass from the two loop level would be of order g^2T . It is worth mentioning that there may be some non-perturbative features that really start from this g^2T scale which makes the computation unreliable beyond g^2 order. It will be also interesting to extend

this method to $3 + 1$ dimensional QCD where the generation of magnetic mass is quite problematic due to IR divergences in the two-loop level.

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Appendix

$$\begin{aligned}
f_R(y_R, y_I) = & \hat{g}^2 \left\{ \frac{2\pi^2}{3} - \pi \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \right\} \\
& + \hat{g}^4 \left\{ 3.7\pi^2 - \frac{2\pi^3}{3} \left[\frac{1}{2\sqrt{y_R^4 + y_I^4}} + \frac{y_R^2}{2(y_R^4 + y_I^4)} \right]^{\frac{1}{2}} - b_1 \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \right. \\
& - b_2 y_R^2 + \left(\pi + \frac{2\pi^2}{3} \right) \ln(y_R^4 + y_I^4) - \frac{1}{2} b_3 y_R^2 \ln(y_R^4 + y_I^4) + b_3 y_I^2 \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) \\
& - 7.9 \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} - \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} + \frac{\pi}{2} y_I^2 - \frac{\pi}{6} \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \ln(y_R^4 + y_I^4) \\
& \left. - \frac{\pi}{3} \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) + \frac{\pi}{3} \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) \ln(y_R^4 + y_I^4) \right\} \\
& + 0(\hat{g}^5) \tag{A.1}
\end{aligned}$$

$$\begin{aligned}
f_I(y_R, y_I) = & \hat{g}^2 \left\{ -\pi \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} - \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \right\} \\
& + \hat{g}^4 \left\{ \frac{2\pi^3}{9} + \frac{2\pi^3}{3} \left[\frac{1}{2\sqrt{y_R^4 + y_I^4}} - \frac{y_R^2}{2(y_R^4 + y_I^4)} \right]^{\frac{1}{2}} - b_1 \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} - \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} - b_2 y_I^2 \right. \\
& + \left(2\pi + \frac{4\pi^2}{3} \right) \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) - \frac{1}{2} b_3 y_I^2 \ln(y_R^4 + y_I^4) - b_3 y_R^2 \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) - \frac{\pi}{2} y_R^2 \\
& + 7.9 \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} + \frac{\pi}{6} \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} + \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \ln(y_R^4 + y_I^4) \\
& - \frac{\pi}{3} \left[\frac{1}{2} \sqrt{y_R^4 + y_I^4} - \frac{1}{2} y_R^2 \right]^{\frac{1}{2}} \tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) - \frac{\pi}{12} \left(\ln(y_R^4 + y_I^4) \right)^2 \\
& \left. + \frac{\pi}{3} \left(\tan^{-1} \left(\frac{y_I^2}{y_R^2} \right) \right)^2 \right\} \\
& + 0(\hat{g}^5) \tag{A.2}
\end{aligned}$$

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Figure caption

Fig. 1. Graphs contributing to the two point function. Solid cross and solid dot denote vertices for renormalisation counterterms of ϕ^2 and ϕ^4 type respectively, while a dashed cross represents the additional ϕ^2 insertion.

